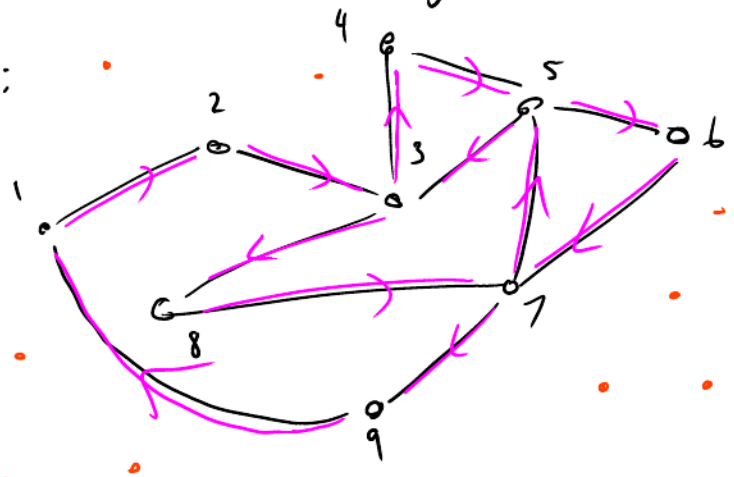
Last Time: Walks in G.

↳ Euler trails.

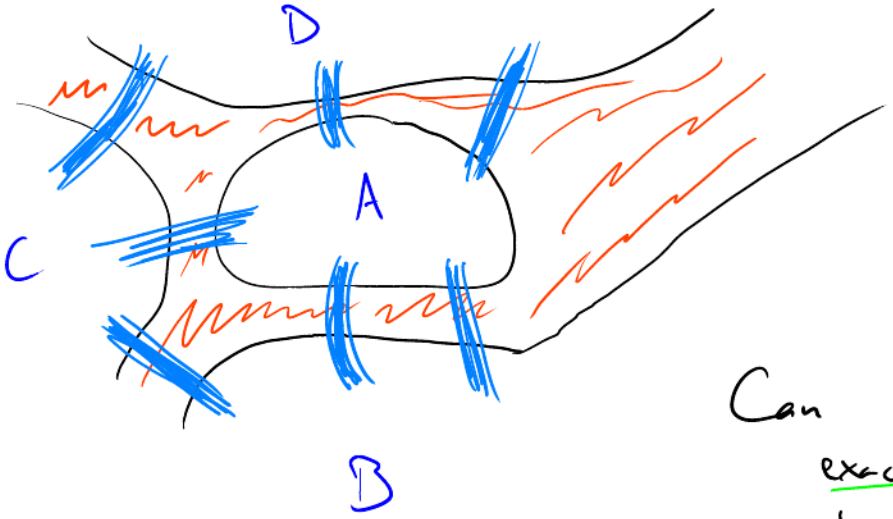
Prop: A graph G has an Euler Trail if and only if at most one component of G has any edges and at most two vertices of G have odd degree.

Ex:



$T = (1, 2, 3, 4, 5, 6, 7, 5, 3, 8, 7, 9, 1)$ is a closed Euler trail in G.

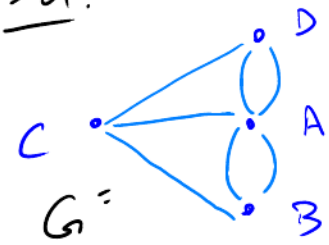
Ex: Suppose a city has this map:



Each bridge crossing the rivers dividing the city's land mass has a memorial statue.

Can we visit all statues exactly once by crossing the bridges on foot?

Sol:



is the graph describing the map:

Land masses become vertices, bridges become edges

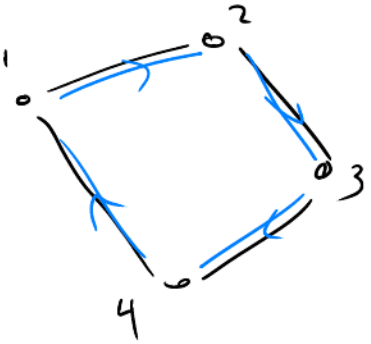
$\deg(C) = 3 = \deg(D) = \deg(B)$, so G does NOT have an Euler trail. Thus there is no way to make the requested path.

□

Hamilton Paths

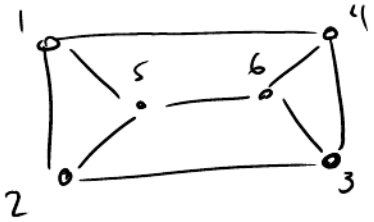
Defn: A graph G has a **Hamilton path** P when P visits every vertex of G exactly once. There is a **Hamilton cycle** in G when there is a Hamilton path P in G such that the initial and terminal vertices of P are adjacent in G.

Ex:



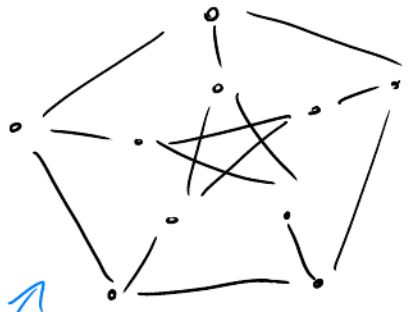
$(1, 2, 3, 4, 1)$ is a Hamilton cycle in C_4.

Ex:



$(1, 2, 3, 4, 6, 5, 1)$ is a Hamilton cycle

Ex:



$= P$

Does P have a Hamilton cycle?

Very hard...

Point: Existence of Euler trails is easy: just check all the degrees. Existence of Hamilton cycles (or even paths) is much more difficult.